A Fundamental Revision of Wind Turbine Design Theory
Mansberger Blade Element Theory
M-BET

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Larry Mansberger
Mansberger Aircraft Inc.
758 Aviator Dr.
Fort Worth, Texas 76179
Ph. 817-439-5978

The following paper contains a discussion of theory which leads into the design methodology and descriptions of a new wind turbine configuration for which a patent is pending. This paper is being self-published in the spirit of a free flow of information. Amazingly in the age of the internet, the author was surprised to find the research for the paper hindered by both stringent academia and restrictive professional journals. Peer review and discussion were found to be lacking in much of the work studied. In that regard, the author welcomes comments, corrections and discussion at wind.turbine.mai@gmail.com.

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Larry Mansberger
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A fundamentally revised theory of wind turbine design and analysis is presented. With respect to conventional theory, assumptions, mistakes and misconceptions that have become ingrained into the historic theoretical basis of wind turbine design have been disproven and removed. New solutions for the momentum and energy equations are derived which align better with natural observation and empirical data. A thermodynamic model of the wind turbine wake is developed based on equations for isentropic flow. The resulting conclusions challenge old theory and propose new wind turbine configurations which could alter the direction of the industry.

Nomenclature

A	=	area
a	=	axial induction factor, $a = (V_1\text{-}V_2)/\ V_1$
a_i	=	inflow velocity ratio, $a_i = V_2/\ V_1 = (1\text{-}a)$
B	=	number of blades
b	=	axial slipstream factor, $b = (V_1\text{-}V_6)/\ V_1$
b_i	=	outflow velocity ratio, $b_i = V_6/V_1 = (1\text{-}b)$
c	=	chord, local blade
c_d	=	drag coefficient for a section, uppercase for a surface
c_l	=	lift coefficient for a section, uppercase for a surface
c_p	=	specific heat at constant pressure
c_v	=	specific heat at constant volume
e	=	energy per unit mass
h	=	enthalpy per unit mass
K	=	ratio of change in enthalpy to ke
k	=	specific heat ratio c_p/c_v
D	=	drag, $D = qC_DA$
F	=	force
ke	=	kinetic energy per unit mass
L	=	lift, $L = qC_LA$
M	=	momentum
$\dot{m}$	=	mass flow $\dot{m} = \rho VA$
P	=	power
p	=	pressure
q	=	dynamic pressure $q=\rho V^2/2$
R	=	maximum radius or gas constant in equation of state
r	=	local or relative radius of blade element
T	=	thrust or temperature

V	=	velocity, with subscript defining location
W	=	work or energy
$\dot{W}$	=	power, work per unit time
α	=	airfoil relative angle of attack
θ	=	pitch angle of rotor blade
λ	=	tip speed ratio, $\lambda = \Omega R/V_1$, λ_R may also be used
λ_r	=	local speed ratio, $\lambda_r = \Omega r/V_1$
λ_s	=	slipstream speed ratio, $\lambda_s = \omega r/V_1$
λ_S	=	slipstream outer speed ratio , $\lambda_S = \omega R/V_1$
ρ	=	density
σ	=	solidity, $\sigma = Bc/2\pi r$
τ	=	torque
ϕ	=	relative flow angle
Ω	=	angular velocity of turbine
ω	=	angular velocity of slipstream

Subscripts

e	=	exit or energy per unit mass
i	=	initial or inlet
n	=	normal to turbine disc.
r	=	with respect to an annular element located at radius r
θ	=	with respect to the direction of rotation, tangentially
rel	=	relative to the airfoil

I. Introduction

Modern wind turbine design has challenged my intuition since I first viewed with awe a large scale modern wind turbine. I can sit endlessly and watch their beautiful high performance composite wings rotating in the breeze. Their design presents for me a personal challenge. Modern composite aircraft design has been a long time passion of mine. From my early days of building and flying hang gliders, sailplanes and experimental aircraft, I have come to realize that I am cursed with the trait that I can't leave well enough alone and must always try to make things better. When I sit and watch wind turbines turn I see missed opportunity in the passing of every turbine blade. This has set me on a mission to better understand the roots of current theory while looking forward with an open mind to the next generation of wind turbines. This paper presents my interpretation and insight into modern wind turbine theory which I refer to as Mansberger Blade Element Theory (M-BET).

Modern wind turbine output has been increasing with the use of faster turning, larger diameter rotors on higher towers, all pushing the limits of practical manufacturing, transportation and construction. At the same time the potential of smaller distributed energy sources beckons for new and better solutions. In almost a century, little has changed within the theory used to design wind turbine configurations or the algorithms used to optimize the number of blades or planform. This is due in part to assumptions, mistakes and misconceptions that have become ingrained into the historical theoretical basis of wind turbine design.

My research leads me to the realization that before M-BET theory most modern wind turbine design and analysis has been based on the work of Hermann Glauert. His most referenced work, entitled "Airplane Propellers" was published in 1935, notably after his death and without his final proof reading. Although thorough, well written and including summaries of work from other classical geniuses of his day, it contains only a small portion devoted to wind turbines, 17 out of 191 pages. After studying all and with great respect for his work, I think his coverage of wind turbines was probably only in the interest of completeness in covering "airscrew theory" of all types. After all one must consider the times, purpose and funding of his work. This was during a buildup in technology between World Wars. The focus was on increasing performance of high speed aircraft and rotorcraft; wind turbine design would have been of little concern. By Glauert's time in history the multi-bladed water pumping wind turbine was for the most part already empirically perfected and fossil fuels were readily available for power generation. Many small electrical generating wind turbines of propeller type blade design as well as large and small multi-bladed electrical turbines were being manufactured previous to and independent of Glauert's work. A focused interest on wind turbines as a source for alternative energy would not occur until the early 1970's. In my opinion it was mere convenience that Glauert's blade element momentum theory quickly became the basis from which most modern wind turbines would be designed and built then and through present time.

Computational fluid dynamics (CFD) is obviously used in much present day analysis. Colorful graphic depictions of streak lines from blade tips look impressive but do not accurately predict the entire flow field. CFD does not determine the equations of fluid motion, momentum or energy and can only do the intensive work of evaluating extremely numerous calculations among the immediately adjoining interdependent equations of fluid elements, the Navier-Stokes equations. The complexity of wind turbine design still requires that sound theory first determine the equations that influence the broader flow field and the complex interrelationship between it and the fluid elements. Only then can an overall analysis and understanding of turbine design and optimal configurations be carried out. As well, computer-processing-time prevents examining all of the numerous design iterations that are possible. To quote Peter Jamieson [11] "current CFD techniques do not yet solve the Navier Stokes equations with the same objectivity as mother nature… It is through a mixture of techniques and convergence of insights coupled with experimental feedback that progress is made."

I assume as I write this paper that the readers are familiar with conventional wind turbine design theories and basic blade element analysis, if not I refer you to the excellent references[2,3,4] listed at the end of this paper. My goal is to challenge these current theories while expanding on important principles and offering up my own design theories. Although it seems redundant to reproduce here many of the details of conventional theory, I will summarize some of them in order to clarify what I significantly disagree with and to point out the errors in the underlying assumptions and misconceptions.

II. Fundamental Concepts

Let's begin by defining some terms and station positions for identifying the flow variables in the mathematical model. Caution, this numbering system varies from conventional numbering in order to more precisely describe the wake and regions near the turbine. Referring to Fig. 1 the airflow through the turbine is commonly represented as a stream tube. At this point we will make no commitment to the shape of this stream tube either upstream or downstream, understanding that this shape will be a result of the turbine design and conditions of operation. Station 1 represents the initial position of influence and station 6 the final position of influence. Stations 2 and 3 represent positions just forward and aft of the turbine respectively. Stations 4 and 5 are reserved for later discussion. Subscripts 0 and ∞ will be reserved for total and free stream conditions respectfully with $V_1 = V_\infty$.

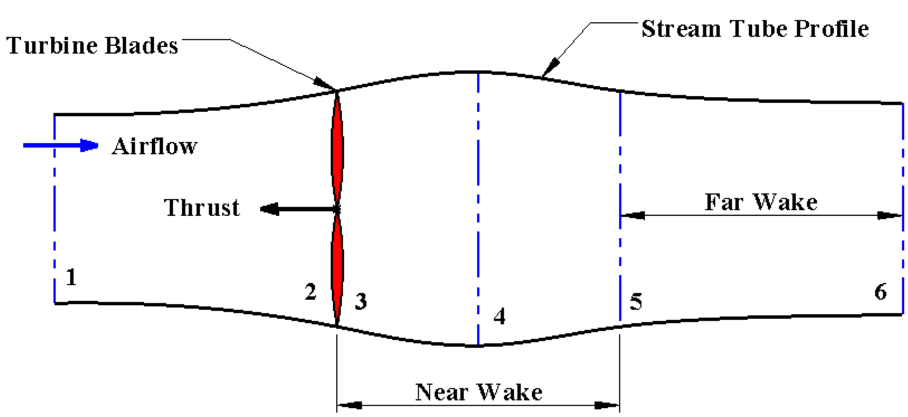

Figure 1 Station Positions

Convention defines axial induction factor ***a*** as the ratio of induced flow velocity, the decrease in inflow velocity, over the free stream velocity or $a = (V_1 - V_2)/ V_1$. In order to simplify future equations I will define a more convenient term, the inflow velocity ratio as $a_i = V_2 / V_1 = (1-a)$. Glauert also uses a term called the axial slipstream factor which for a wind turbine would be defined as $b = (V_1 - V_6)/ V_1$. I will similarly define outflow velocity ratio as $b_i = V_6 / V_1 = (1-b)$.

Before going any further let's review the fundamental difference between a propeller and a wind turbine. The purpose of a propeller is to convert power from an aircraft engine equal to shaft torque τ times angular velocity Ω into thrust T, or more accurately the power to be converted into F_n V, normal force times velocity reacting with the free stream. The purpose of a wind turbine is to convert power from the free stream wind into torque times angular velocity to be converted into electrical energy or another form of useful work. Currently most all conventional wind turbine designs use rotational motion to extract power from the wind. The power extracted from the wind or the aircraft engine is therefore equal to

$$P = \tau \cdot \Omega \tag{1}$$

The torque applied to the wind turbine must be equal and opposite of the subsequent torque reacting with the slipstream. The rotation or angular velocity of the slipstream however can and will be different from that of the turbine blades. Therefore we will carefully distinguish between Ω defined as the angular velocity of the turbine or propeller from ω, the angular velocity of the slipstream or an annular element of the slipstream.

The rotational kinetic energy per unit mass of air contained in an annular element of the slipstream is equal to

$$ke_{\theta r} = \frac{1}{2}\omega^2 r^2 \tag{2}$$

where r is equal to the radial position of the element versus R, which is reserved for the maximum radius of the turbine. The purpose of the aircraft propeller is to produce thrust through a change in linear momentum or in other words to accelerate the air downstream preferably without this rotation. If the propeller also causes the slipstream to rotate then this is an inefficiency which wastes the aircraft power by producing the unnecessary rotation. Conventional wind turbine theory has mistaken this to be a similar condition of the wind turbine. However, with the rotating wind turbine we cannot extract power from the wind unless we do impart an equal and opposite torque and therefore rotation into the slipstream. The only way to eliminate this rotation would be through the use of stator vanes before or after the rotating turbine or with counter rotating blades. But this is currently not the case and may not be necessary as we shall see the rotating air mass may be to our advantage increasing the performance of the wind turbine. The common misconception is that for a given power extraction it is more efficient to have Ω high with τ and ω low, but since Ω and τ are not independent parameters this condition may not be that of maximum power extraction. Although there are structural advantages to low torque there are disadvantages to high tip speed rotation in the form of increased drag losses, centrifugal forces, noise and the probability of bird strikes.

Let's look at the equation for the power output of the wind turbine which is derived from Euler's turbine equation which looks like

$$P = \dot{m}\Omega(r_i V_{\theta,i} - r_e V_{\theta,e}) = \dot{m}\,\Omega\, r V_{\theta,e} \tag{3}$$

where V_θ represents the tangential velocity in the direction of rotation which at our turbine inlet is assumed to be zero. At the exit of an annular element V_θ will be equal to ωr. Mass flow, $\dot{m}$ is equal to ρVA, ρ the density of the air, $V=V_2=a_i V_1$ the velocity of the air through the turbine. For the power contained in an infinitesimal annular element, A is the area equal to $2\pi rdr$ at the turbine. We can now put the elemental power equation in the following form

$$dP = 2\pi\rho\, a_i V_1 \Omega\, \omega\, r^3 dr \tag{4}$$

It can clearly be seen from this formula that power extracted increases with an increase in slipstream rotation ω equally important to blade rotation Ω. Notably the most prominent variable in the equation is the radius at which the power extraction occurs.

It is premature at this point to make the assumption carried over from propeller theory that for a given power a high rotor angular velocity Ω will be more efficient because less energy will be wasted into the slipstream in the form of rotational kinetic energy, ke_θ. This assumption again is in error because ω and Ω are complicated functions of one another and not independent variables. More precisely $\omega = f(a_i, \Omega, V, B, r, c, c_l, c_d)$, where c is the airfoil chord, c_l and c_d are the airfoil lift and drag coefficients respectfully and B is the number of blades. To further complicate it, the inflow velocity ratio is also a function of a similar set of variables $a_i=f(\Omega,B,r,c,c_l,c_d)$. We can simplify these functions by defining the parameters: solidity as $\sigma=Bc/2\pi r$, the tip speed ratio $\lambda=\Omega R/V_\infty$ the local speed ratio $\lambda_r=\Omega r/V_\infty$, and defining a new term slipstream rotation speed ratio $\lambda_s=\omega\, r/V_\infty$.

Convention defines the non-dimensional power coefficient Cp as the ratio of power extracted to the theoretical power within the kinetic energy of the air stream or

$$C_P = P\Big/\left(\frac{1}{2}\rho V_1^3 A\right) \tag{5}$$

Alternately, we can define Cpi as the non-dimensional power coefficient for an individual annular element with the use of the elemental power equation Eq. (4) and dividing by the elemental area $2\pi rdr$ yields

$$C_{Pi} = \frac{2\pi\rho\, a_i V_1 \Omega \omega r^3 dr}{\left(\rho V_1^3 \pi\, rdr\right)} \tag{6}$$

Canceling terms, rearranging, and using our newly defined parameters simplifies to

$$C_{Pi} = 2a_i\left(\frac{\Omega r}{V_1}\right)\left(\frac{\omega\, r}{V_1}\right) = 2a_i\lambda_r\lambda_s\,. \tag{7}$$

I present this relationship, $C_{Pi} = 2a_i\lambda_r\lambda_s$ as being the most fundamental relationship available for understanding the design and analysis of wind turbines. Keep in mind though that none of these terms are independent variables that can be decoupled.

Nothing here is yet controversial, just presented from a slightly different perspective with no assumptions. We will come back to this fundamental equation as I develop the new theory. Bottom line is we can't make any assumptions or generalizations about the power formula until we know exactly what these functions ω and a_i are equal to. The basis for M-BET is to derive formulas for ω and a_i directly from the conditions of operation and the airfoil aerodynamics. In order to do so we will first need to fully understand the mathematical relationships at each station of the turbine as they apply to the energy and momentum equations and then we can work towards solutions for b_i, a_i, $\boldsymbol{\omega}$, and the thrust and power equations. But before continuing on we need to discuss more the inherent flaws in conventional theory.

III. Challenging Current Theory

A. Glauert Corrected

Glauert correctly identified the rotational energy terms within propeller design theory. If we divide Eq. (4) by $\dot{m} = \rho VA$ the mass flow through the annular element, it yields the energy extracted or induced into the airflow per unit mass and appears in terms of the rotational parameters

$$e_{out} = \frac{2\pi\rho\, a_i V_1 \Omega \omega r^3 dr}{\left(\rho\, a_i V_1 2\pi\, rdr\right)} = \Omega\omega r^2 \tag{8}$$

And we have the previously mentioned rotational kinetic energy in the slipstream

$$ke_{\theta r} = \frac{1}{2}\omega^2 r^2 \tag{9}$$

From these Glauert deduced a change in pressure head across the propeller due to rotational terms of

$$\Delta p = \rho\,\Omega\omega r^2 - \frac{1}{2}\rho\,\omega^2 r^2 = \rho\left(\Omega - \frac{1}{2}\omega\right)\omega r^2 \qquad (10)$$

This seems logical, but since this change in pressure occurs across the propeller or turbine this equation is not correct with respect to the way pressure changes with the energy equation to be discussed later. For a propeller the difference is inconsequential since $\Omega >> \omega$ and the thrust term is really the primary importance in propeller blade element design. But from here, Glauert arbitrarily defines a term, a' as the rotational interference factor also called the tangential induction factor.

$$a' = \frac{\omega}{2\Omega} \quad \text{or} \quad \omega = 2a'\Omega \qquad (11)$$

He uses this equation for the convenience of relating power and thrust formulas to induction factors simplifying the pressure differential of a propeller to

$$\Delta p = 2\rho\,\Omega^2(1-a')a' r^2 \qquad (12)$$

For a propeller equation Eq. (11) and Eq. (12) make sense. From Eq. (11) we see the angular velocity of the slipstream is proportional to the angular velocity of the propeller and in the same direction, from Eq. (12) at the same time the slipstream rotation is seen to detract from the pressure differential induced by the propeller. Glauert later uses this same term a' in wind turbine design[1]. For use in the design of a wind turbine he simply states that "*the thrust, torque, and interference factors will all be negative*."[1] But this statement is not completely accurate; for a wind turbine ω is in the opposite direction of Ω, but for a fixed pitch blade it is also inversely proportional. If we remove some of the load from the turbine and allow it to accelerate in the same wind velocity, then as the rotation Ω increases the absolute value of ω decreases until the blade is slipping with no applied torque and ω = 0. If we add power and torque in the opposite direction the propeller state is induced and ω reverses direction. Furthermore if blade angles are optimal when Ω=0 then ω is at a maximum negative value for the wind turbine; although not extracting any power a fixed non-rotating turbine blade will cause the slipstream to rotate at a maximum angular velocity in the opposite direction of a propeller. So in fact Eq. (11) and Eq. (12) are not valid equations for use in wind turbine design.

Conventional theory uses Glauert's relationship $\omega = 2a'\Omega$ to incorrectly optimize the wind turbine based on maximizing the power formula relative to induction factors a and a' decoupled from Ω. This is invalid as none of these terms are independent variables that can be decoupled, a' just like ω is equal to $f(a_i, \Omega, V, B, r, c, c_l, c_d)$. In addition $\omega = 2a'\Omega$ is used to incorrectly solve for the airflow triangle at the turbine blades. Although sufficient for the design of a propeller, Glauert's formula for a' does not accurately represent the wind turbine state and yet this definition has incorrectly become much of the basis for optimizing modern wind turbine designs. The effect of this mistake is to incorrectly devalue the importance of the ω term in the optimization of the wind turbine power equation.

B. Misuse of 1-D Momentum and Bernoulli's Equations

Most conventional theories and versions of Blade Element Momentum method, BEM are in some manner based on the misuse of both simple one dimensional momentum theory and Bernoulli's equation. For a thorough explanation of "1-D Momentum Theory", I refer you to Martin Hansen's *Aerodynamics of Wind Turbines*[3]. This book covers the subject in easy to understand detail, enough to unknowingly expose the gross error in its fundamental derivation. The underlying and questionable assumptions for 1-D Momentum Theory are an ideal wind turbine modeled by a semi-permeable disc being acted on by a one dimensional ideal fluid flow which is inviscid (frictionless), incompressible and with no wake rotation. As previously discussed, if there is no wake rotation there is no power being extracted from the conventional turbine. Furthermore 1-D momentum theory is usually used to evaluate internal pipe flow and is not necessarily relevant to free stream air flow as we shall see. But ignoring all this, the theory is still important in predicting the thrust and axial induction factor of the turbine if we properly correct for these poor assumptions.

In its simplest form the momentum equation follows from Newton's 2nd law that the sum of the forces acting on a body is equal to its change in momentum.

$$\sum F = \frac{d(mV)}{dt} = \dot{m}(V_{out} - V_{in}) \qquad (13)$$

In this case we are dealing not with a body but a control volume of air. Hansen offers up not one but both of the accepted control volume diagrams which often accompany the standard derivations. These are redrawn here in Fig. 2 and Fig. 3 using our nomenclature and station positions.

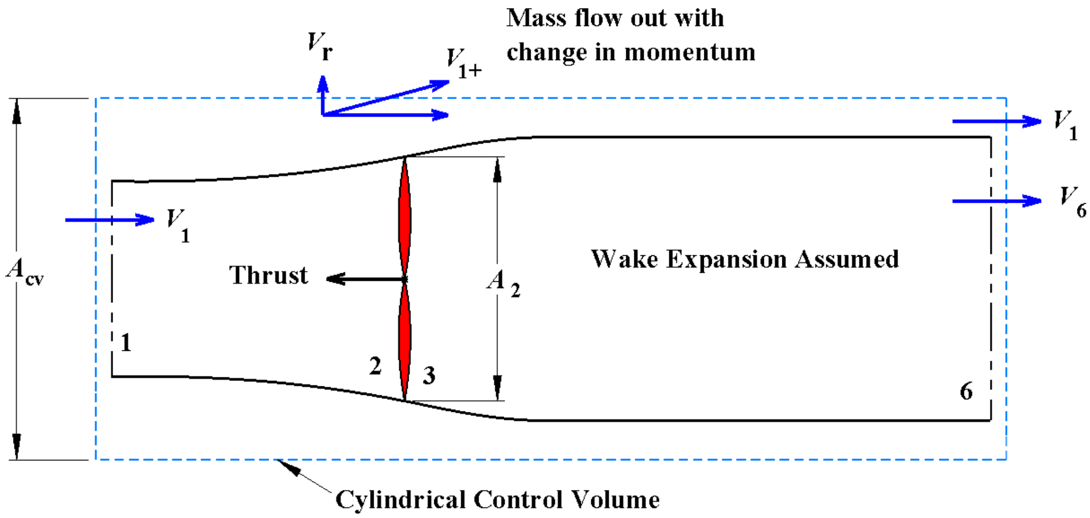

Figure 2 Conventional CV I

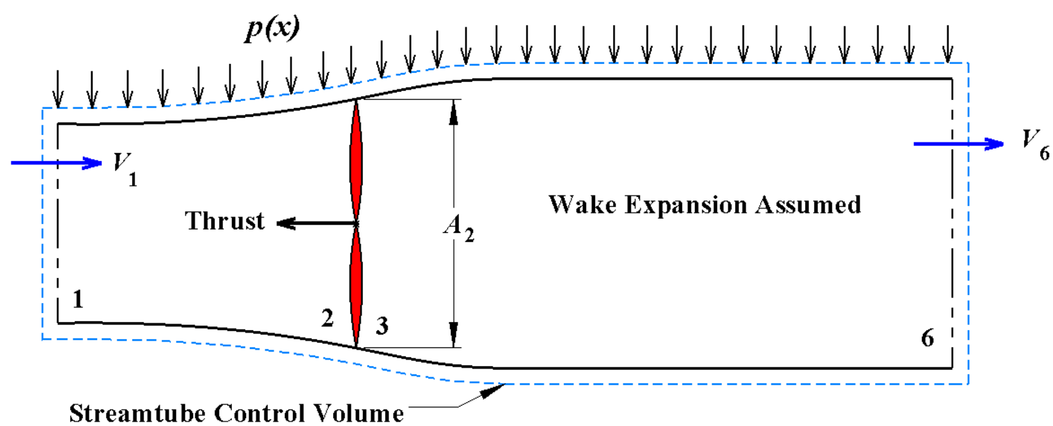

Figure 3 Conventional CV II

The first control volume diagram Fig. 2, accounts for the flow both through the turbine and at some dimension external to it parallel with the freestream airflow. Therefore as the airflow is deflected around the turbine some mass flow and momentum must exit out the side of the control volume, which immediately contradicts the assumption of 1-D flow. In the first derivation, the control volume diagram is given the following equation:

$$\rho V_6^2 A_6 + \rho V_1^2 \left(A_{cv} - A_6\right) + \dot{m}_{side} V_1 - \rho V_1^2 A_{cv} = -T \tag{14}$$

Furthermore the change in momentum out the side is incorrectly assumed to be its mass flow times the velocity of the air stream V_1. But if the airflow is exiting the control volume as it approaches the turbine at varying stream tube diameters then at each elemental station a portion will exit at a different velocity. In this case a different speed as well as angle, implying a V_x as well as V_y term or more correctly a radial term V_r.

The conservation of mass is used to calculate the mass flow out the side as

$$\dot{m}_{side} = \rho A_1 \left(V_1 - V_6\right) \tag{15}$$

From here the thrust or drag on the turbine becomes

$$T = \rho V_2 A_2 \left(V_1 - V_6\right) = \dot{m}\left(V_1 - V_6\right) \tag{16}$$

In the alternative control volume Fig. 3 the flow is strictly considered only within the streamlines which pass through the turbine. The sum of forces must be integrated along the pressure distribution of the outer streamline surface. This corrected control volume diagram yields:

$$T = \dot{m}\left(V_1 - V_6\right) - F_{pressure} \tag{17}$$

This equation is correct. But conventional theory compares Eq. (16) to Eq. (17) and declares that $F_{pressure}$ must equal zero. The correct declaration should have been that the derivation of Eq. (16) is oversimplified and incorrect. Again, the airflow in Eq. (16) exits the control volume at differing radial velocities as it approaches the turbine. This causes a change momentum which when combined with shear forces in the velocity gradient must balance with the changing static pressure at each elemental x station. The integration of these pressures will not be equal to zero but will equal some positive value resulting from a complicated nonlinear relationship acting on the control volume along the perimeter of the outer streamline. These calculations are currently beyond myself and this paper, although they should be considered significant enough to discount the validity of basic 1-D momentum theory.

The next step in conventional theory is to reconcile the results from this momentum equation with Bernoulli's Equation. A further foundation for conventional theory is the misuse of Bernoulli's equation to relate the pressure drop across the turbine to the energy extracted and change in kinetic energy far down stream. Even though Bernoulli's equation is invalid across any device which changes the total energy of the flow, conventional theory incorrectly carries this out by comparing the equations up steam and down steam of the turbine as follows

$$\textit{Up Stream} \quad p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \tag{18}$$

$$\textit{Down Stream} \quad p_3 + \frac{1}{2}\rho V_3^2 = p_6 + \frac{1}{2}\rho V_6^2 \tag{19}$$

But even these equations are invalid as written. We defined stations 1 and 6 as the limits of the turbines influence. The flow in these regions is not an internal pipe flow but is the result of the turbines influence and therefore Bernoulli's Equation does not apply.

Conventional theory carries on regardless with Bernoulli's Equation, assuming that the airflow is a constant density or incompressible and at constant temperature. Therefore since mass flow must be conserved across the turbine $V_3 = V_2$. If Δp is defined as $\Delta p = p_3 - p_2$ then $p_3 = p_2 + \Delta p$. Furthermore it is assumed the final pressure far down stream equals the original static pressure $p_6 = p_\infty$. Eq. (19) then becomes

$$p_2 + \Delta p + \frac{1}{2}\rho V_2^2 = p_\infty + \frac{1}{2}\rho V_6^2 \tag{20}$$

Solving Eq. (18) and Eq. (20) for Δp yields

$$\Delta p = \frac{1}{2}\rho\left(V_6^2 - V_1^2\right) \tag{21}$$

This result, Eq. (21) is an incorrect solution in contradiction to the energy equation. Among the flaws was defining $\Delta p = p_3 - p_2$. The energy extraction occurs between station 2 and 3, and pressure cannot change without a change in temperature which negates Bernoulli's premise. We will come back to and elaborate on this later.

The thrust or drag on the turbine is then assumed to be equal to this pressure drop times the area of the turbine and this is equated to Eq. (16).

$$\frac{1}{2}\rho\left(V_1^2 - V_6^2\right) A_2 = \rho V_2 A_2 \left(V_1 - V_6\right) \tag{22}$$

Solving Eq. (22) for V_2 yields

$$V_2 = \frac{1}{2}\left(V_1 + V_6\right) \tag{23}$$

or in terms of velocity ratios

$$a_i = \frac{1}{2}\left(b_i + 1\right) \tag{24}$$

This is a fundamental and incorrect premise of conventional theory that the velocity through the turbine is equal to the average of the free stream and final velocity of the affected airflow. This is derived based on ignoring slipstream rotation and in conjunction with misuse of the linear momentum and Bernoulli's equations. Even Glauert admits this is not accurate stating, "*the conclusion obtained…that the axial velocity u (V_2) at the propeller disc is the arithmetic mean of the axial velocity V_1 and the slipstream velocity u_1 (V_6) is not true in general when rotation also occurs in the slipstream*." Glauert[1]. So if we re-assume that slipstream rotation must be important for wind turbine performance we need to derive a corrected solution.

But let's carry on with the conventional solution for now. Solving for b_i yields

$$b_i = 2\,a_i - 1 \tag{25}$$

Conventional theory in Eq. (25) implies that if the inflow velocity ratio is one half of the free stream velocity then the final velocity will equal zero. This by conservation of mass flow implies an expansion of the wake to an infinite diameter at an inflow velocity ratio of 0.5, which by applying any common sense would be a physical impossibility. This is an accepted limitation of the 1-D Momentum Theory that it is invalid as it approaches axial induction factors of 0.5. It is often said to be associated with the condition of a turbulent wake state, but I would have to disagree. Within this derivation there is no set up for predicting turbulent versus laminar flow around the turbine. I contend this solution is simply invalid overall. Nature may be able to be defined by some mathematical relationship, but a given mathematical relationship does not necessarily define a phenomenon of nature. Air does not flow in this manner, or as Einstein allegedly said, "Your calculations are correct, but your grasp of physics is abominable."

From here conventional theory calculates thrust and power coefficients based on the previous errors. Equation (23) is used to calculate V_6 in terms of axial induction factor yielding

$$V_6 = V_1(1 - 2a) \tag{26}$$

Thrust is calculated entering this value into Eq. (16) yielding

$$T = 2\rho a(1-a)V_1^2 A_2 \tag{27}$$

By definition thrust coefficient is calculated by dividing through by dynamic pressure and area $\frac{1}{2}\rho V_1^2 A_2$, yielding

$$C_T = 4a(1-a) \tag{28}$$

This relationship is graphed in Fig. 9. Notice that this result peaks at a = 0.5 and goes to zero as the axial induction factor goes to 1.0. This does not occur in nature and makes no sense under any circumstances including turbulent wakes.

C. Betz Debunked

Power in conventional theory is calculated either by the mass flow times the deficit in kinetic energy or from thrust times velocity, both conveniently resulting in a common solution when it is assumed that the pressure drop is equal to the change in *ke*. Omitting the derivation and again using axial induction factor as the parameter the result is

$$P = 2\rho V_1^3 A_2 a(1-a)^2 \tag{29}$$

Calculating power coefficient by definition dividing through by $\frac{1}{2}\rho V_1^3 A_2$ leaves

$$Cp = 4a(1-a)^2 \tag{30}$$

Taking the derivative of this and setting equal to zero gives the pre-1920 result known as Betz's limit which claims the maximum C_p attainable is 16/27 at an axial induction factor of 1/3.

The next assumption that was in error here is that power calculations are a function of $F_n V$, thrust times velocity at the turbine disc. This is not a propeller and even if it was there is no direct connection mechanically or mathematically between this term and the power extracted term, $P = \tau \cdot \Omega$. $F_n V$ is the power which turbine is causing to be transferred from kinetic energy to internal energy. $F_n V$ is also not to be confused with the power lost to drag. Drag losses over the airfoil reduce both thrust and rotation. The effect of thrust from the wind turbine is to reduce the momentum and increase the internal energy of the airflow; it is not directly tied to the energy extraction. I will cover this in depth when we get to the energy equation.

I further reiterate, as shown, these previously accepted results are based on an incorrect solution to the momentum equation and misuse of Bernoulli's equation to solve an energy problem. The results are neither a law nor a valid limit and serve only to stifle innovation. In addition power extraction is not a result from the reduction in kinetic energy. If the conservation of mass flow requires that the velocity of airflow entering the turbine actuator disc equals the velocity exiting the turbine then the conventional theory that the wind turbine extracts power from kinetic energy of the wind by slowing it down makes little sense. The energy transfer occurs at the turbine with an assumed negligible change in velocity. Therefore in fact kinetic energy at this point is the medium through which the energy is transferred, but the power in a rotating wind turbine is extracted from the differential pressure across the airfoil which is being converted to negative thrust and torque, but only the torque results in energy extraction. The relevant energy equations must include the rotational terms and are invalid if temperature is ignored because the makeup of the internal energy of the flow is changing. To increase turbine performance we must increase mass flow while dropping both the temperature and pressure simultaneously. We will come back to this. Again the most relevant equation for power extracted from the conventional wind turbine is $\tau\Omega$, torque times angular velocity and the driving equation $C_{Pi} = 2a_i\lambda_r\lambda_s$.

IV. Developing New Theory

A. Corrected Momentum Equation

So what does the corrected momentum equation look like? We must first start with an accurate control volume diagram that works regardless of any assumptions about wake expansion. Let's start with a control volume Fig. 4 that assumes the streamlines flowing around the turbine and coming back together behind it. This is what we know usually occurs in natural disturbances to airflow.

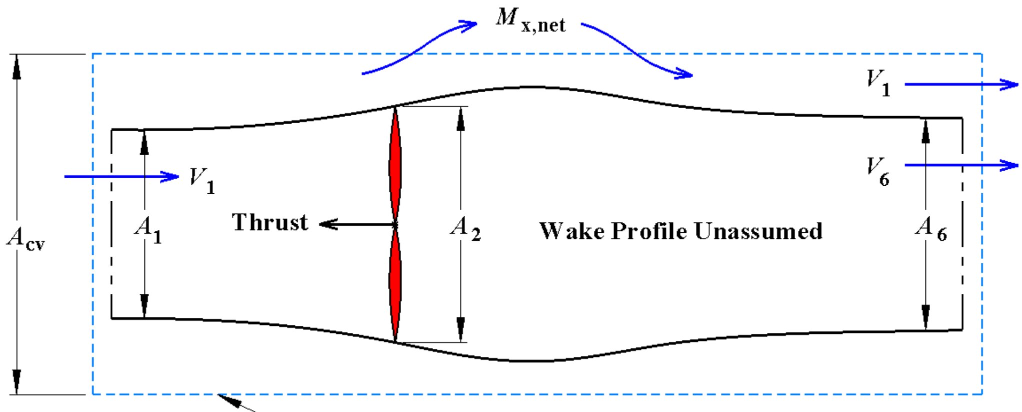

Figure 4 MBET Momentum CV.

In control volume diagram Fig. 4, I make no assumptions about the velocity or mass flow representing the change in momentum out the sides. At this point we will just call it $M_{x,net}$. Summing the forces and equating to the change in momentum through the control volume gives us the following

$$-T = -(\rho V_1 A_{cv})V_1 + M_{x,net} + (\rho V_6 A_6)V_6 + \rho V_1^2 (A_{cv} - A_6) \tag{31a}$$

Simplifying we arrive at the corrected momentum equation of

$$T = \rho A_6 \left(V_1^2 - V_6^2\right) - M_{x,net} \tag{31b}$$

Although we don't have a direct analytical solution for $M_{x,net}$ we can arrive at an estimated solution by deductive analysis. Examining the limits of the equation, we know that when $V_1 = V_6$ the flow through the turbine must be unrestricted with zero change in momentum, zero thrust and therefore $M_{x,net(V_1=V_6)} = 0$. When $V_6 = 0$ then we have reached a state of total restriction through the turbine so all the mass flow and momentum must be flowing out around the front of turbine and back in behind the turbine. So what is the limit of the thrust and $M_{x,net(V_6=0)}$ for this condition? For turbulent wake states we can consider the turbine drag may be similar to the drag on a flat plate or disc. This is usually referred to in a non-dimension form as coefficient of drag C_D or wind turbine designers like to refer to it as coefficient of thrust C_T (In aviation thrust opposes drag, in wind energy thrust acts in the same direction as drag), either way

$$C_T = C_D = \frac{(T\,or\,D)}{qA_2} \quad \text{where} \quad q = \frac{1}{2}\rho V_1^2 \tag{32}$$

Depending on the Reynolds number this value can vary from 1.2 to 1.8 for a turbulent wake. But for our limit we want to imagine maintaining the flow as laminar around the turbine. In this case the pressure on the front face of the turbine would be $+q$ and on the back side $-q$ for a total drag force of $2qA_2$ or $C_T = 2$. So for $V_6 = 0$ and $T = 2qA_2 = \rho V_1^2 A_2$ the momentum equation becomes

$$\rho V_1^2 A_2 = \rho A_6 V_1^2 - M_{x,net(V_6=0)} \tag{33}$$

Solving for $M_{x,net(V_6=0)}$

$$M_{x,net(V_6=0)} = \rho V_1^2 (A_6 - A_2) \tag{34}$$

If $V_6 = 0$, then obviously $A_6 = 0$ not ∞ and this equation gives us the result we expect. $M_{x,net(V_6=0)} = \rho V_1^2 A_2 = 2qA_2$. Using the continuity equation (conservation of mass flow) $V_2 A_2 = V_6 A_6$ we can manipulate Eq. (34) to a final form of

$$M_{x,net(V_6=0)} = \rho V_1^2 A_2 \left(\frac{V_2}{V_6} - 1 \right) = 2qA_2 \left(\frac{V_2}{V_6} - 1 \right) \tag{35}$$

Looking back at our previous limit $M_{x,net(V_1=V_6)} = 0$, the continuity equation also implies if there is no restriction to airflow then $V_1 = V_2 = V_6$. Inserting this into Eq. (35) coincidentally also equals zero. This might imply that Eq. (35) is a general equation approximated by the linear relationship shown in Fig. 5.

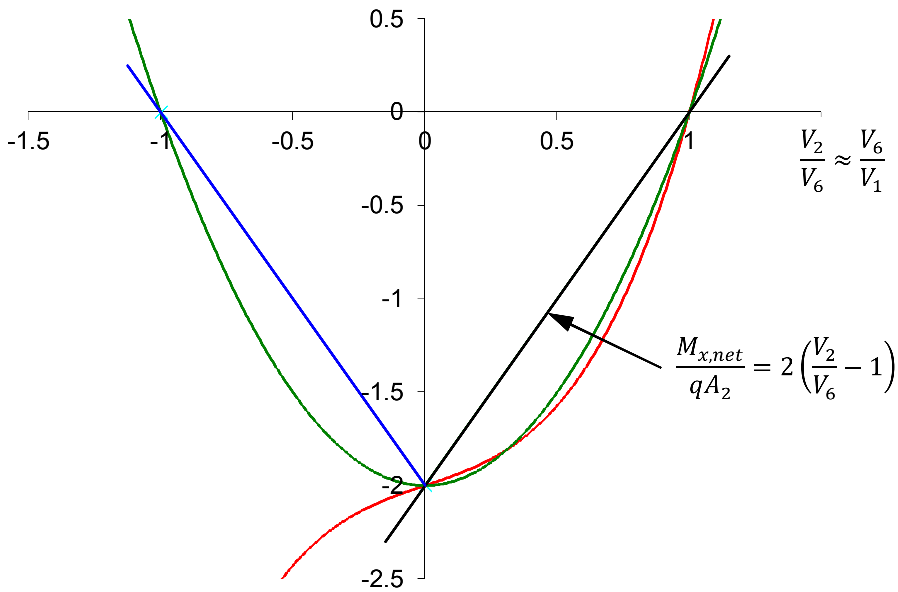

Figure 5 Possible $M_{x,net}$ Solutions

In Fig. 5 increasing negative values of V_2/V_6 and $M_{x,net}$ would be produced by a propeller brake state with energy being added to reversing the airflow through the turbine. The limit for the wind turbine state would be $V_2/V_6 = 0$. Reversed flow caused by negative values of V_6 would produce the blue negative sloping line. This condition could only be produced in an open throat wind tunnel with flow coming from both directions, with mass flow and momentum flowing out of the control volume parallel to the turbine disc and when $V_6 = -V_2 = -V_1$ then $M_{x,net} = 0$. The intersection of the lines would represent the condition where $A_1 = A_6 = 0$, with no flow through the turbine and a theoretical laminar wake. Possible quadratic or cubic solutions are additionally shown in green and red.

What if we now look at $M_{x,net}$ as a function of the non-dimensional $V_6/V_1 = b_i$. We know that if there is no restriction $V_1 = V_2 = V_6$ then $b_i = 1$ and $M_{x,net} = 0$. We also assume that if $V_6 = 0$ then $b_i = 0$ and $M_{x,net} = -2qA_2$. Let's further assume another point that if $V_6 = -1$ which means $b_i = -1$ implying reverse flow in the wake as above, then again we have $M_{x,net} = 0$. These points all match the points plotted in Fig. (5) for V_2/V_6.

Admittedly we do not know for sure that these are linear relationships; they could be quadratic or cubic, and may be differing? But we do know based on the corrected momentum equation and within the range of the wind turbines flow parameters the relationship exists that

$$M_{x,net} \approx 2qA_2 \left(\frac{V_2}{V_6} - 1 \right) \approx 2qA_2 \left(\frac{V_6}{V_1} - 1 \right) \tag{36}$$

and observing from this

$$\frac{V_2}{V_6} \approx \frac{V_6}{V_1} \tag{37}$$

If correct Eq. (37) has profound implications for the momentum equation and changes the way we analyze the airflow in the wake of a wind turbine. Rearranging terms yields

$$\frac{V_2}{V_1} = \left(\frac{V_6}{V_1}\right)^2 \quad \text{or} \quad a_i = b_i^2 \tag{38}$$

or more useful

$$b_i = \sqrt{a_i} \tag{39}$$

Fig.6 graphs the new result $b_i = \sqrt{a_i}$ compared with the conventional $b_i = 2\,a_i - 1$ in terms of $a=1-a_i$.

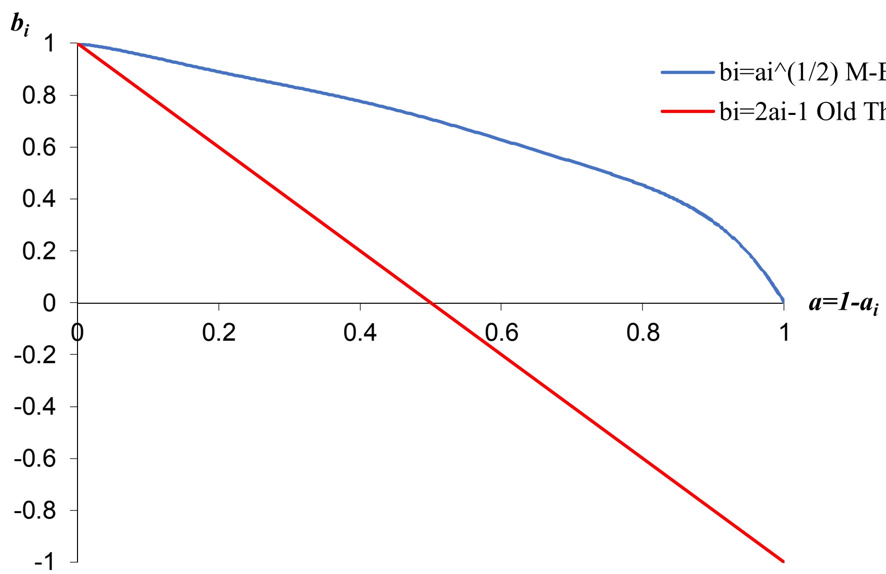

Figure 6 Relationships of b_i vs. axial induction, $a=1-a_i$

It should be obvious that $b_i = \sqrt{a_i} = \sqrt{(1-a)}$ is a much more realistic solution. The M-BET formula does not break down at axial induction factors above 0.5. Note that conventional theory claims that $b_i < a_i$; from conservation of mass flow the wake therefore would be expanding as the velocity in the wake is decreasing. MBET claims the opposite that $b_i > a_i$ and the wake therefore must be contracting as its velocity is increasing. Which is correct? Natural observations would agree with the later. I also refer you to not one but two independent sources of empirical data from Doppler radar experiments, Figure 7 and Fig. 8 from Ref. 7 and Ref. 8 respectfully. The data collected and presented in these papers is truly enlightening. Both sources clearly show that after the initial decrease in velocity through the turbine the velocity increases downwind. Figure 7 clearly shows cross sections of the contracting wake with increasing velocity. Figure 8 graphically depicts the velocity deficit. From figure 8 we can approximate a_i equal to 1 minus the velocity deficit at the turbine or 1-0.36 = 0.64. Similarly b_i = 1 minus the downstream final velocity deficit or 1-0.18 = 0.82. Inserting into Eq. (39) and comparing $b_i = \sqrt{a_i}$, $b_i = \sqrt{0.64} = 0.80$, which is amazingly close and supports the new theory.

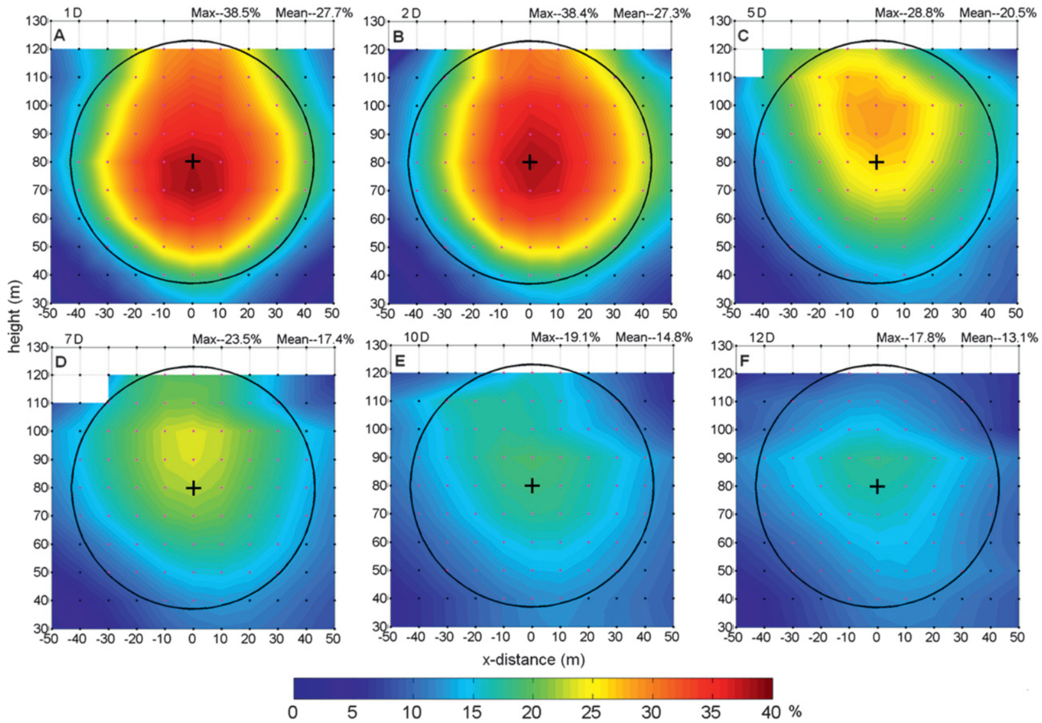

Figure 7 Doppler Radar Data; Hirth B.D. and Schroder J.L.[5]

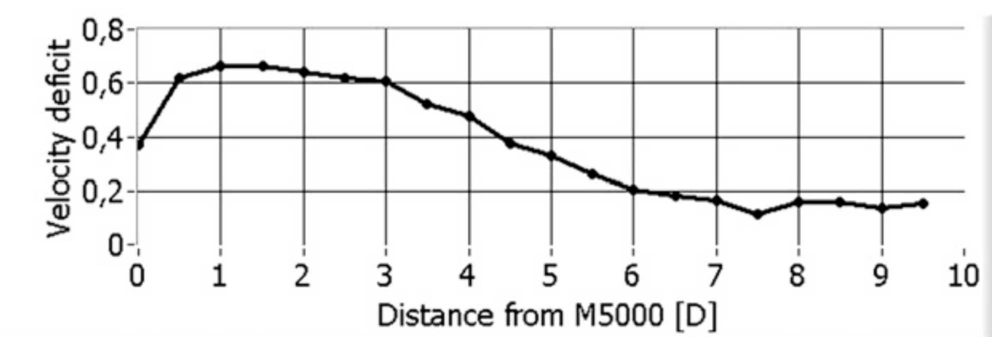

Figure 8 Doppler Radar Data; Kasler, Y., Rahm, S. and Simmet, R.[6]

Continuing with the new theory, we can now take our deductive solution Eq. (35) and insert back into Eq. (31) giving us the corrected momentum equation,

$$T = \rho A_6 \left(V_1^2 - V_6^2\right) - \rho V_1^2 A_2 \left(\frac{V_6}{V_1} - 1\right) \tag{40}$$

Dividing through be qA_2 we arrive at the non-dimensional

$$C_T = 2\left[\frac{A_6}{A_2}\left(1 - \frac{V_6^2}{V_1^2}\right) - \left(\frac{V_6}{V_1} - 1\right)\right] \tag{41}$$

From the continuity equation $V_2 A_2 = V_6 A_6$ therefore $\frac{A_6}{A_2} = \frac{V_2}{V_6} = \frac{a_i}{b_i}$ and with $b_i = a_i^{1/2}$ we insert all into Eq. (41) arriving at what I like to call the laminar wake momentum equation.

$$C_T = 2\left(1 - a_i^{3/2}\right) \tag{42}$$

There is additional empirical support for the validity of the M-BET laminar wake momentum Eq. (42). Glauert and others collected and supposedly corrected early wind tunnel data to arrive at a graph originally depicted by Eggleston and Stoddard[4] in 1987. A version of that graph appears as Fig. (9).

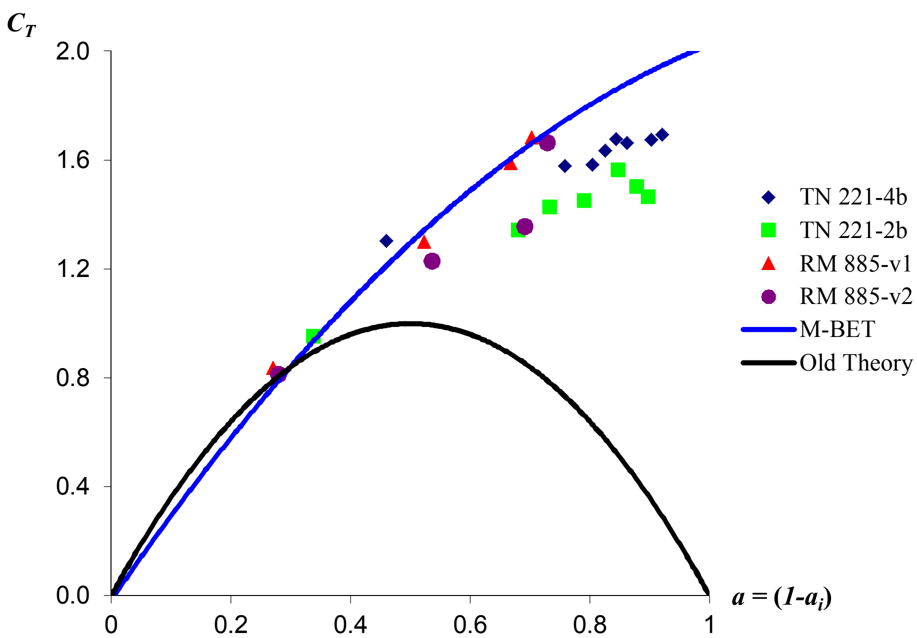

Figure 9 Axial Induction Factor vs. C_T for Empirical Data

Figure 9 clearly shows a far superior fit of the data by the M-BET laminar wake equation then by any other historic alternative. Even empirical and modern formulas such as the recent technical report by the National Renewable Energy Lab[7] fail to properly correct the relationship between a_i and b_i. Most importantly the Laminar Wake Equation explains this and shows extreme accuracy in the regions where wind turbines most probably need to be operating in the mid-regions of the graph. Error in some of the data fit may be due to data reduction and interpretation, wind tunnel effects or a turbulent wake increasing turbine back pressure.

Continuing with the development of the new theory we solve Eq. (42) for inflow velocity ratio

$$a_i = \left(1 - 0.5C_T\right)^{2/3} \tag{43}$$

Equation (43) defines a_i the first fundamental parameter needed for the MBET algorithm.

B. The Correct Energy Equation

Now let's look at the true and correct energy equation. It is undisputed in any fluid or thermo-dynamic textbook that if the flow between two regions contains a mechanical device such as a propeller or a wind turbine, Bernoulli's equation is not valid. There are no exceptions, this is very important. Obviously this applies between stations 2-3. As previously stated we also can't analyze between stations 1-2 or 3-6 independently of 2-3 using Bernoulli's equation in the currently accepted manner. Why? Because the most significant term in the energy equation is the internal energy related to ΔT the change in temperature. (Note from here on out the variable T will be used exclusively for temperature and for thrust we will use F_n the force normal to the turbine.) Conventional theory ignores ΔT by assuming temperature before and after the turbine is the same and that the only transfer of energy is from pressure energy independent of temperature. But the true energy equation does not allow the pressure to change independent of temperature.

There are many forms of the general energy equation for dealing with flow through a turbine. For an ideal gas ignoring rotation, the simplest form of the energy equation is

$$c_p T_2 + \frac{1}{2}V_2^2 = c_p T_3 + \frac{1}{2}V_3^2 + e_{out} \tag{44}$$

The term c_p is the specific heat of the fluid at constant temperature. The value of this term for air is often sited at 0.240 Btu/lb°R. Be careful, this needs to be converted to ft^2/s^2°R for consistent units in the above equation. The conversion is approximately 1 Btu/lb°R = 25,037 ft^2/s^2°R the c_p constant at 77°F works out to be equal to approximately 6008. This gives one an idea of the significance of the internal energy of the system versus the kinetic energy. Further notice that according to Eq. (44) if we extract energy from the flow then the velocity cannot remain constant across the turbine without a temperature drop.

For an ideal gas $c_p T$ can be considered equal to enthalpy h, which is tied to pressure and temperature in the following equation

$$c_p T = h = \frac{p}{\rho} + c_v T \tag{45}$$

where c_v also must be converted from its commonly used 0.171 Btu/lb°R to approximately 4,291 ft^2/s^2°R. Equation (45) into Eq. (44) yields

$$\frac{p_1}{\rho_1} + 4291 \cdot T_1 + \frac{1}{2}V_1^2 = \frac{p_3}{\rho_3} + 4291 \cdot T_3 + \frac{1}{2}V_3^2 + e_{out} \tag{46}$$

The above again demonstrates the thermal energy potential in the air flow.

The enthalpy relationship between temperature and pressure can be related by the ideal gas equation of state formula $p = \rho R T$, where R in this case is the gas constant for air equal to 1716.51 ft^2/s^2/°R. Furthermore the relationships between specific heats and the gas constant are related by the following relationships:

$$R = \left(c_p - c_v\right) \text{ and } k = {}^{c_p}\!/\!{}_{c_v} \text{ ,where for air k = 1.4} \tag{47}$$

Using the above, the energy equation between stations 1-6 can be manipulated to look like this,

$$3.5\frac{p_1}{\rho_1} + 0.5V_1^2 = 3.5\frac{p_6}{\rho_6} + 0.5V_6^2 + e_{out} \tag{48}$$

A solution to this equation would result in the correction to Eq. (21), but as can be seen the result is dependent on a change in density and temperature with no straight forward solution, at least not without considering the thermodynamics.

So let's refocus right now on the basic energy equation between stations 1-6 and see what we can deduce. This equation similar to Eq. (44) appears as

$$c_p T_1 + \frac{1}{2}V_1^2 = c_p T_6 + \frac{1}{2}V_6^2 + e_{out} \tag{49}$$

and

$$e_{out} = c_p\left(T_1 - T_6\right) + \frac{1}{2}\left(V_1^2 - V_6^2\right) \tag{50}$$

This is energy per unit mass, in order to solve for power out we multiply by mass flow and rearrange terms,

$$P = \rho_2 V_2 A_2 [c_p (T_1 - T_6) + \frac{1}{2}(V_1^2 - V_6^2)] \tag{51}$$

Dividing through by $\frac{1}{2}\rho_1 V_1^3 A_2$, with $\rho_1 \approx \rho_2$ and still governed by the momentum equation from Eq. (38) $b_i^2 = a_i$ will give us the dimensionless power coefficient,

$$Cp = \frac{2a_i c_p (T_1 - T_6)}{V_1^2} + a_i - a_i^2 \tag{52}$$

Let us now define the ratio in the change in enthalpy to kinetic energy available as *K*

$$K = \frac{c_p (T_1 - T_6)}{\frac{1}{2}V_1^2} \tag{53}$$

Inserting Eq. (53) into Eq. (52) yields

$$Cp = (K+1)a_i - a_i^2 \tag{54}$$

Equation (54) is the corrected version of Eq. (30) which Betz used to derive his supposed limit. If we take the derivative with respect to a_i ,set equal to zero and solve

$$\frac{dCp}{da_i} = K + 1 - 2a_i = 0 \tag{55}$$

$$a_{i,optimum} = \frac{1}{2} + \frac{K}{2} \tag{56}$$

$$Cp_{\max} = \frac{K^2 + 2K + 1}{4} \tag{57}$$

This solution implies the only limiting factor to our power extraction is *K*, our change in enthalpy relationship. If we make the conventional assumption that the temperature is constant then *K*=0 which implies $Cp = a_i - a_i^2$. Taking the derivative of this, setting equal to zero and solving we get a maximum *Cp* of 0.25 at an inflow velocity ratio and or axial induction factor in this case of 0.5. This is far below Betz's supposed limit and occurring at the velocity ratio where conventional theory breaks down. But this is considering the kinetic energy contribution only.

I am not the first to suggest that the kinetic energy contribution could be this low. See references (10) *A Modified Form of the Betz' Wind Turbine Theory Including Losses A.* Dyment 1989, (11) *Reformulation of the Momentum Theory Applied to Wind Turbines* by Ricardo Prado 1995 and (12) *Limits of the Turbine Efficiency for Free Fluid Flow* by Gorbon, Gorlov and Silantyev 2001. These papers independently use revised models of the flow field along with more advanced mathematics to derive maximum power coefficients of closer to 0.30 and less. The results of these papers have apparently been ignored, probably because we know that modern 3 blade wind turbines can achieve power coefficients exceeding 0.45. These previous papers should have been a wakeup call to revise the conventional theory, but for the past 25 years we keep hitting the snooze button.

So what does all the above imply? Let's assume that a hypothetical wind turbine with a $Cp = 0.40$ is operating at a supposed optimum design axial induction of 1/3 (inflow velocity ratio of 2/3). We can solve for the theoretical ΔT that would be required for the additional performance. Solving Eq. (52) for ΔT

$$\Delta T = \frac{V_1^2}{2c_p a_i}\left(C_P + a_i^2 - a_i\right) \tag{58}$$

Choosing $Cp = 0.40$, $c_p = 6011$, $a_i = 0.67$ and an example wind speed of say 30ft/s or roughly 20.5 mph,

$$\Delta T = \frac{30^2}{2(6011)(0.67)}\left(0.40 + 0.67^2 - 0.67\right) = 0.02^\circ F$$

In other words it only takes a temperature drop of $0.02°F$ from the enthalpy term to shift the power coefficient from less than 25% to 40%. If we graph Eq. (54) in the figure below, we can see that the a_i, Cp plane or the result of kinetic energy only, is not representative of the turbine limit but merely the turbine kinetic energy baseline curve.

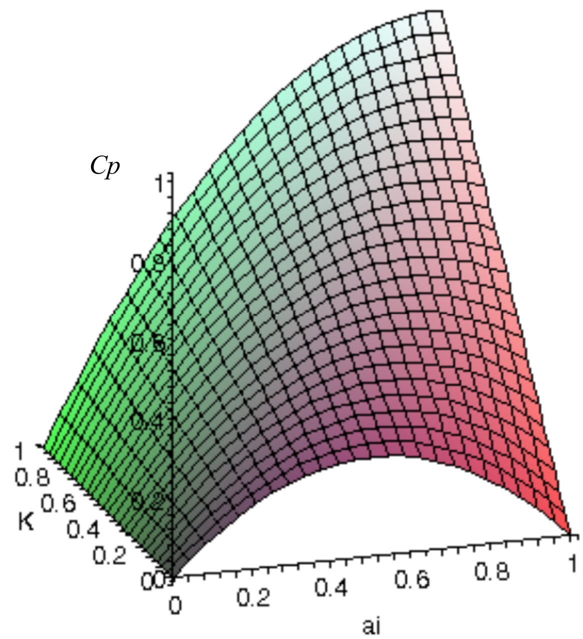

Figure 10 Graph of *Cp* Equation (54)

So we must ask, by what means is this temperature drop and resulting increase in performance accomplished and controlled? In order to understand, we must now put rotational terms into the energy equation. As currently presented the energy equation is still not correct without the critical rotational terms. We return to Eq. (2) for the rotational kinetic energy per unit mass contained in an annular element of slipstream and to Eq. (4) for the power extracted from the same annular element. Dividing Eq. (4) by $\dot{m} = \rho VA$ the mass flow through the annular element, yields the energy extracted per unit mass which appears in terms of the rotational parameters

$$e_{out} = \frac{2\pi\rho\, a_i V_1 \Omega \omega r^3 dr}{\left(\rho\, a_i V_1 2\pi\, r dr\right)} = \Omega \omega r^2 \tag{59}$$

We can now insert Eq. (2) and Eq. (59) into the basic energy equation

$$c_p T_1 + \frac{1}{2}V_1^2 = c_p T_6 + \frac{1}{2}V_6^2 + \frac{1}{2}\omega_6^2 r_6^2 + \Omega\omega_3 r_3^2 \tag{60}$$

Equation (60) is the corrected energy equation which is relevant for the design of any wind turbine which extracts energy through rotation. Rearranging and solving for ΔT

$$\Delta T = T_6 - T_1 = \frac{\left[\frac{1}{2}\left(V_1^2 - V_6^2\right) - \Omega\omega_3 r_3^2 - \frac{1}{2}\omega_6^2 r_6^2\right]}{c_p} \tag{61}$$

This answers the earlier question, how do we induce the temperature drop and increase the performance of the turbine. Equation (61) shows the temperature drop must be a function of the rotational parameters alone. As can be seen decreasing the velocity from station 1 to 6 will actually increase the temperature. This therefore implies the performance enhancement of the turbine must come from increasing the rotational term $\Omega\omega_3 r_3^2$.

Let's look at the term $\Omega\omega_3 r_3^2$ more carefully. This term was derived from the Euler's Turbine equation and as pointed out previously the slipstream rotation appears equally important to the blade rotation. Even more important is the fact that the energy extracted will increase with the square of the radius of the annular element from which the flow passes through. Use of this important fact along with accelerated flow concepts to be discussed later are both central to the design of the next generation of high performance wind turbines. But before getting into new design concepts let's look at this new theory closer.

C. The Thermodynamic Wind Turbine Model

This theory as with most, assumes a laminar wake unaffected by viscous outer layer or turbulent mixing. We have corrected both the momentum equation and the energy equation. This basic wake model is drawn based on observations of naturally occurring flow around obstructions and in agreement with the new momentum equation. For the purpose of better understanding the thermodynamics of the airflow through the turbine, let's examine a realistic hypothetical case study for a micro-wind turbine. We will give it the following parameters and conditions;

Diameter 6 ft., R = 3 ft	V_∞ = 30 ft/s
Area = 28.27 ft^2	p_∞ = 2116.2 lb/ft^2
Cp = 0.40	ρ_∞ = 0.002378 slug/ft^3
$a=1/3$, $a_i=2/3$	λ_R = 6

I apologize for carrying out this paper and case study in the English system of units; I will revise it to the metric system as soon as time allows.

We will not be using any form of Bernoulli's equation; we will use only the true energy equation, the equation of state $p = \rho R T$ and where appropriate the thermodynamic equations for isentropic flow found in any fluid or thermo-dynamic text,

$$\left(\frac{T_f}{T_i}\right)^{\left(\frac{k}{k-1}\right)} = \left(\frac{\rho_f}{\rho_i}\right)^{(k)} = \left(\frac{p_f}{p_i}\right) \tag{62}$$

As noted before, very small changes in temperature, hundredths of a degree can have significant effects on the energy equation. Therefore it is essential to carry as many decimals places as possible throughout the calculations and to use correct gas and specific heat constants that obey the relationships in Eq. (47). The following values were found to give reasonable results,

c_p = 6007.79 ft^2/s^2	$R = (c_p - c_v)$ = 1716.51 ft^2/s^2
c_v = 4291.28 ft^2/s^2	$k = c_p / c_v$ = 1.40000

In practice knowing the temperature and pressure, the equation of state would be used to calculate the density. To start the case study the equation of state formula is first used to make sure our initial temperature coincides with our constants.

$$T_1 = \frac{p_1}{\rho_1 R} = \frac{2116.2}{(0.002378)(1716.51)} = 518.440°R$$

i. Free Spinning Turbine

I stated earlier that the thrust or normal force on the turbine times velocity $F_n V$ was not directly associated with power extraction but instead results in a shift in internal energy or enthalpy. I would like to demonstrate this by first analyzing the sample turbine, free spinning with no energy extraction. The total energy in the flow stream is often referred to by the stagnation enthalpy

$$h_0 = c_p T + \frac{1}{2}V^2 = \frac{p}{\rho} + c_v T + \frac{1}{2}V^2 \tag{63}$$

If no energy is extracted this value must remain constant, either way it can be used to solve for other parameters and cross check values with it along the turbine stations. Station 1 is self-explanatory with the given values:

T_1 = 518.440°R
p_1 = 2116.20 lb/ft^2
ρ_1 = 0.002378 slug/ft^3
V_1 = 30 ft/s
h_{01} = 3115128.6 ft^2/s^2

Given the axial induction factor of 1/3, or inflow velocity ratio of 2/3, the velocity at station 2 has been defined as $V_2 = a_i\ V_1$ = 20 ft/s. Knowing h_0 must remain constant we can use Eq. (63) to solve for T_2 and use Eq. (62) to solve for the rest of the parameters.

T_2 = 518.482°R
p_2 = 2116.7944 lb/ft^2
ρ_2 = 0.00237848 slug/ft^3
V_2 = 20.00 ft/s
h_{02} = 3115128.6 ft^2/s^2

Next, using the laminar wake Eq. (42) we calculate the thrust coefficient.

$$C_T = 2\left(1 - a_i^{3/2}\right) = 2\left(1 - (2/3)^{1.5}\right) = 0.911338$$

The thrust coefficient can be directly correlated to the both the normal force acting on the turbine and the pressure drop between station 2-3 with the following

$$C_T = \frac{F_n}{qA_2} = \frac{-\Delta p_{2,3}}{q} \tag{64}$$

Therefore

$$\Delta p_{2,3} = -qC_T \text{ where } q = \frac{1}{2}\rho V_1^2 \tag{65}$$

For this case

$$q = 0.5(0.002378)(30)^2 = 1.0701 \text{ lb/ft}^2$$

and

$$\Delta p_{2,3} = (1.0701)(-0.911338) = -0.975222 \text{ lb/ft}^2$$

We now have all the information we need to calculate the station 3 parameters. Based on the conservation of mass flow $\dot{m} = \rho VA$ is constant through the turbine. We can make the assumption that $V_2 \approx V_3$ if the change in density is very small and confirm it later. So in order to balance with the momentum equation we know $p_3 = p_2 + \Delta p_{2,3}$ =2.116.2-.97522=2115.819 lb/ft^2. In this first analysis we are assuming no power is extracted therefore $T_2 = T_3$, and ρ_3 can be calculated from the equation of state:

Free Spinning
T_3 = 518.4816°R
p_3 = 2115.8192 lb/ft^2
ρ_3 = 0.00237738 slug/ft^3
V_3 = 20.00 ft/s
h_{03} = 3115128.6 ft^2/s^2

Confirming reasonableness, the mass flow, $\dot{m} = \rho_2 V_2 A_2$ = 1.34478 slug/s. If we solve for $V_3 = \dot{m}/(\rho_3 A_2)$ = 20.009 ft/s, and insert it back in the total energy equation we confirm $V_2 \approx V_3$ and $T_2 \approx T_3$ within 0.0001 degree, insignificant to iterate for more accuracy.

The procedure for solving station 6 is a simple matter of calculating V_6 based on the outflow ratio derived from the momentum equation, $V_6 = b_i V_1 = \sqrt{a_i} V_1$ = 24.4949ft/s. Now plugging V_6 into the total energy Eq. (63) one can solve for T_6. Final pressure p_6 is assumed to be back to atmospheric and final density is calculated from the equation of state.

Free Spinning
T_6 = 518.4650°R
P_6 = 2116.2 lb/ft^2
ρ_6 = 0.00237789 slug/ft^3
V_6 = 24.4949 ft/s
h_{06} = 3115128.6 ft^2/s^2

Notice the final temperature is 0.025°F higher. Because no energy was extracted this is the resulting shift from kinetic to internal energy caused by $F_n V$, thrust times velocity, demonstrating, $F_n V$ is in no way associated with power extraction other than to deter mass flow.

I use the isentropic relations to work backwards from station 6 to the previous station with the same diameter as A_2 and velocity $\approx V_2$ and called this station 5. Because the velocity is the same $T_5 = T_3$ and rearranging the isentropic equations

$$p_5 = \frac{p_6}{\left(\frac{T_6}{T_5}\right)^{3.5}} = \frac{2116.2}{\left(\frac{518.465}{518.482}\right)^{3.5}} = 2116.438$$

Density is again calculated from the equation of state and all the parameters for station 5 are known.

Free Spinning
T_5 = 518.4816°R
P_5 = 2116.4377 lb/ft^2
ρ_5 = 0.00237808 slug/ft^3
V_5 = 20.00 ft/s
h_{05} = 3115128.6 ft^2/s^2

Note that although temperature and velocity are the same for stations 3 and 5, pressure and density are not and there is not an isentropic solution from stations 2 to 5 or for any of the stations in between. This region of flow from station 2-5 does not flow in accordance with the isentropic equations, but must still obey the energy and momentum equations. It was strictly energy and momentum equations that solved for station 3. The difference in pressure between 3 and 5 must be reconciled by nature with a change in momentum and area at station 4. This is similar to the reduction in area of a fluid jet as it exits a pressurized reservoir or in this case exits the influence of the turbine. The pressure must increase from p_3 to p_5 passing through p_∞. The velocity must slow to a minimum and then start increasing steadily to V_6. This can conveniently occur if the area expands to a maximum at station 4 coincident with minimum velocity V_4 and p_∞ atmospheric pressure.

I define station 4 as the region of minimum velocity and maximum wake diameter. To solve for station 4 refer to the two control volume diagrams in Figure 11.

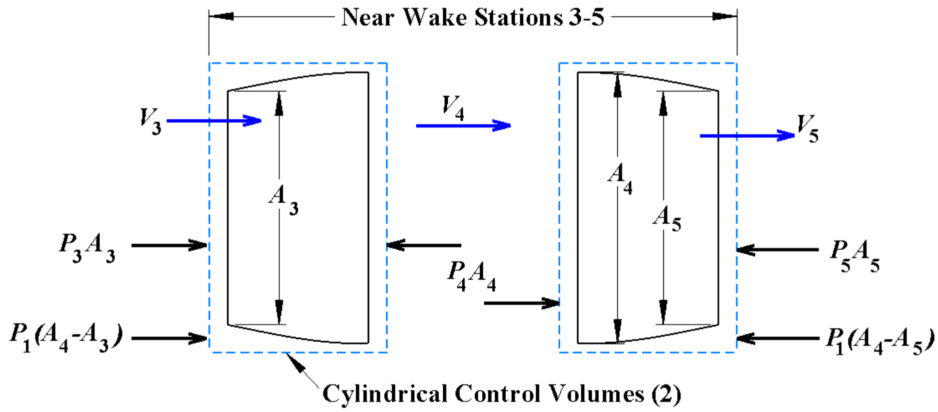

Figure 11 Non Isentropic Wake Region

Assuming the angle of flow to be small on both sides of station 4, then the sum of the forces around each CV diagram must be approximately equal to the change in momentum which yields the following two equations

$$p_3A_3 + p_\infty(A_4 - A_3) - p_4A_4 = \dot{m}\,(V_4 - V_3) \tag{66}$$

$$p_4A_4 - p_5A_5 - p_\infty(A_4 - A_5) = \dot{m}\,(V_5 - V_4) \tag{67}$$

Assuming $p_4 = p_\infty$, and by the definition of station 5 we have $A_3 = A_5 = A_2$ and as well $V_3 \approx V_5 \approx V_2$, then these equations can be simplified to

$$p_3A_2 - p_\infty A_2 = \dot{m}\,(V_4 - V_2) \tag{68}$$

$$p_\infty A_2 - p_5A_2 = \dot{m}\,(V_2 - V_4) \tag{69}$$

Now subtracting the bottom equation from the top and solving for V_4 yields

$$V_4 \approx \frac{(p_3 + p_5 - 2p_\infty)A_2}{2\dot{m}} + V_2 \tag{70}$$

This equation (70) reconciles the momentum and energy equations between stations 3-5 and determines the parameters at station 4. Entering values

$$V_4 \approx \frac{(2115.8192 + 2116.4377 - 2(2116.2))28.27}{2(1.3445)} + 20 \approx 18.5 \text{ ft/s}$$

This continued reduction in velocity after leaving the turbine is due to both the continued influence of the turbine as well as the radial momentum of the mass flow. The later may not be adequately accounted for in this theory but it does give us agreement with the characteristic flow. Conservation of mass flow requires an increase in the wake diameter to a maximum at this point. This is only a temporary expansion and is observed in the Doppler radar data of Fig. 8.

Continuing with the station 4 solutions, plugging V_4 into the total energy equation one can solve for T_4. Final pressure p_4 is assumed to be passing through atmospheric p_∞ and final density is calculated from the equation of state.

Free Spinning
$T_4 = 518.4864°R$
$P_4 = 2116.20$ lb/ft^2
$\rho_4 = 0.00237779$ slug/ft^3
$V_4 = 18.50$ ft/s
$h_{04} = 3115128.6$ ft^2/s^2

ii. Energy Extraction

That completed the solutions for the free spinning turbine. We will now solve for the case with energy being extracted. Solutions for stations 1 and 2 will be identical to those for the free spinning case. Now let's look in depth at the nature of the energy being extracted between stations 2-3. Again I choose a realistic hypothetical Cp = 0.40 and $\lambda_R = 6$. By definition of Cp we know the average energy extracted per unit mass flow

$$e_{out} = \frac{C_P q A_2}{\dot{m}} = \frac{C_P 0.5\rho V_1^3 A_2}{\rho V_2 A_2} = \frac{C_P V_1^2}{2a_i} \tag{71}$$

$$e_{out} = \frac{0.4(30)^2}{2(2/3)} = 270 \text{ ft}^2/\text{s}^2$$

We now need an equation which relates what the values of λ_R and λ_S would be for a given overall power output. We previously derived Eq. (6) and Eq. (7) for Cp_i.with λ_r and λ_s. We will now integrate Eq.6 from 0 to the outer radius R.

$$Cp = \frac{\int_0^R 2\pi\rho a_i V_1 \Omega \omega r^3 dr}{\int_0^R \rho V_1^3 \pi r dr} = \frac{\frac{1}{2}\pi\rho a_i V_1 \Omega\omega R^4}{\frac{1}{2}\rho V_1^3 \pi R^2} = a_i\left(\frac{\Omega R}{V_1}\right)\left(\frac{\omega R}{V_1}\right)$$

$$Cp = a_i \lambda_R \lambda_S \tag{72}$$

Inserting Eq. (72) into Eq. (71)

$$e_{out} = \frac{1}{2}V_1^2 \lambda_R \lambda_S = \frac{1}{2}\omega\Omega R^2$$

$$\text{or } \lambda_S = \frac{2e_{out}}{\lambda_R V_1^2} \tag{73}$$

We can now examine the relationship between λ_R and λ_S for these conditions

$$\lambda_S = \frac{2e_{out}}{\lambda_R V_1^2} = \frac{2(270)}{6(30)^2} = 0.1$$

or in terms of radial velocity

$$\omega = \frac{V_1 \lambda_S}{R} = \frac{30(.1)}{3} = 1.0 \text{ rad/s} = 9.5 \text{ rpm}$$

and compared to

$$\Omega = \frac{V_1 \lambda_R}{R} = \frac{30(6)}{3} = 60 \text{ rad/s} = 573 \text{ rpm}$$

Note the turbine blades are spinning 60 times faster that the rotating slipstream.

We need the above information so we can calculate the rotational kinetic energy per unit mass flow exiting the turbine. This can be derived by integrating (Eq.2) from 0 to R and dividing by A_2 or by considering the rotational kinetic energy in a solid cylinder of mass flow, either way yielding

$$ke_{\theta R} = \frac{1}{4}\omega^2 R^2 \tag{74}$$

and for our case study

$$ke_{\theta R} = \frac{1}{4}(1)^2(3)^2 = 2.25 \text{ ft}^2/\text{s}^2$$

Interesting note; comparing this to the rotational energy extracted (2.25/270) = 0.84%, or 1/120th, which implies very little inefficiency at this point in the rotating air mass. A much larger inefficiency occurs in $c_p \Delta T$ = (6007.79)(0.025) = 150 ft^2/s^2 which went into heating the air mass.

We now have the necessary information for the station 3 energy equation with rotational parameters included,

$$h_{02} = c_p T_3 + \frac{1}{2}V_3^2 + \frac{1}{2}\omega\Omega R^2 + \frac{1}{4}\omega^2 R_2 \tag{75}$$

Rearranging to solve for T_3

$$T_3 = \frac{h_{02} - \frac{1}{2}V_3^2 - \frac{1}{2}\omega\Omega R^2 - \frac{1}{4}\omega^2 R^2}{c_p} \tag{76}$$

Inserting our known values with $V_2 \approx V_3$

$$T_3 = \frac{3115128.6 - 0.5(20)^2 - 270 - 2.25}{6007.79} = 518.4363 \text{ °R}$$

Now that we have extracted energy, $h_{02} \neq h_{03}$, but $h_{03} = h_{02} - e_{out}$

$$h_{03} = 3115128.6 - 270 = 3114858.6 \text{ ft}^2/\text{s}^2$$

So the temperature and stagnation enthalpy were determined from the energy equation but the pressure p_3 must still be in balance with the momentum equation. Since we are still assuming the same axial induction factor and thrust coefficient then p_3 must still equal $p_2 + \Delta p_{2,3}$ which equals 2115.819 lb/ft^2. The density is determined from the equation of state fully defining station 3.

Energy Extracted	Free Spinning
T_3 = 518.4363°R	T_3 = 518.4816°R
p_3 = 2115.8192 lb/ft^2	p_3 = 2115.8192 lb/ft^2
ρ_3 = 0.00237759 slug/ft^3	ρ_3 = 0.00237738 slug/ft^3
V_3 = 20.00 ft/s	V_3 = 20.00 ft/s
h_{03} = 3114858.6 ft^2/s^2	h_{03} = 3115128.6 ft^2/s^2

Note in this case we now have a temporary drop in temperature at the turbine of 0.045°F.

The procedure for solving for T_6 is as before using V_6 from the momentum equation in the new total energy equation or

$$T_6 = \frac{h_{03} - \frac{1}{2}V_6^2 - \frac{1}{4}\omega^2 R^2}{c_p} \tag{77}$$

Inserting values into Eq. (77) yields

$$T_6 = \frac{31148586 - 0.5(24.4949)^2 - 2.25}{6007.79} = 518.4196$$

Again final pressure p_6 is assumed to be back to atmospheric and final density is calculated from the equation of state.

Energy Extracted	Free Spinning
T_6 = 518.4196°R	T_6 = 518.4650°R
p_6 = 2116.2 lb/ft^2	p_6 = 2116.2 lb/ft^2
ρ_6 = 0.00237809 slug/ft^3	ρ_6 = 0.00237789 slug/ft^3
V_6 = 24.4949 ft/s	V_6 = 24.4949 ft/s
h_{06} = 3114858.6 ft^2/s^2	h_{06} = 3115128.6 ft^2/s^2

Now with energy extracted we have our previously estimated net temperature drop of 0.02°F

Notice that in this analysis of station 6, I do not distinguish a difference between ω_3 or $\omega_{4,5,6}$. An attempt could be made to analyze this difference based on either a balance in pressure with centrifugal forces or conservation of angular momentum and to account for the effect of the theoretical wake diameter on one or the other. But we have succeeded in this theory on estimating the value of ω_3 as being very small and assume any change downwind in rotational kinetic energy to be insignificant in calculating the slipstream parameters and not to have an effect on overall performance in this case. As rotational kinetic energy is increased in new designs this assumption may have to be reevaluated.

Continuing our analysis with station 5 we again solve by working backwards with the isentropic relationships with $V_5 = V_3$ and $T_5 = T_3$.

$$p_5 = \frac{p_6}{\left(\frac{T_6}{T_5}\right)^{3.5}} = \frac{2116.2}{\left(\frac{518.4196}{518.4363}\right)^{3.5}} = 2116.4386$$

Energy Extracted	Free Spinning
T_5 = 518.4363°R	T_5 = 518.4816°R
P_5 = 2116.4386 lb/ft^2	p_5 = 2116.4377 lb/ft^2
ρ_5 = 0.00237829 slug/ft^3	ρ_5 = 0.00237808slug/ft^3
V_5 = 20.00 ft/s	V_5 = 20.00 ft/s
h_{05} = 3114858.6 ft^2/s^2	h_{05} = 3115128.6 ft^2/s^2

We start the station 4 solution with Eq. (70) inserting new values

$$V_4 \approx \frac{(2115.8192 + 2116.4386 - 2(2116.2))28.27}{2(1.3445)} + 20 \approx 18.5 \text{ ft/s}$$

The difference in V_4 between the free spinning versus energy extracting case is noted as insignificant. We can calculate T_4 in the same manner as T_6, $p_4 = p_\infty$, and density is calculated with the equation of state.

Energy Extracted	Free Spinning
T_4 = 518.4365°R	T_4 = 518.4864°R
P_4 = 2116.20 lb/ft^2	p_4 = 2116.2 lb/ft^2
ρ_4 = 0.00237802 slug/ft^3	ρ_4 = 0.00237779 slug/ft^3
V_4 = 18.5 ft/s	V_4 = 18.5 ft/s
h_{04} = 3114858.6 ft^2/s^2	h_{04} = 3115128.6 ft^2/s^2

This completes the analysis of the thermodynamic wind turbine model. Summarizing, it was done in two cases, free spinning and energy extracting to demonstrate that the thrust force alone determines the turbine wake profile and does not contribute to energy extracted. Also a theoretical difference in the near wake and far wake regions is identified. The far wake like the forward region of influence obeys the isentropic relationships. The near wake is identified as a region of non-isentropic flow. The thermodynamic properties are determined from both the change in momentum due to the thrust and the rotational energy extracted from and imparted into the slipstream. The resulting parameters for the energy extracting case are graphed in the following Fig. 12.

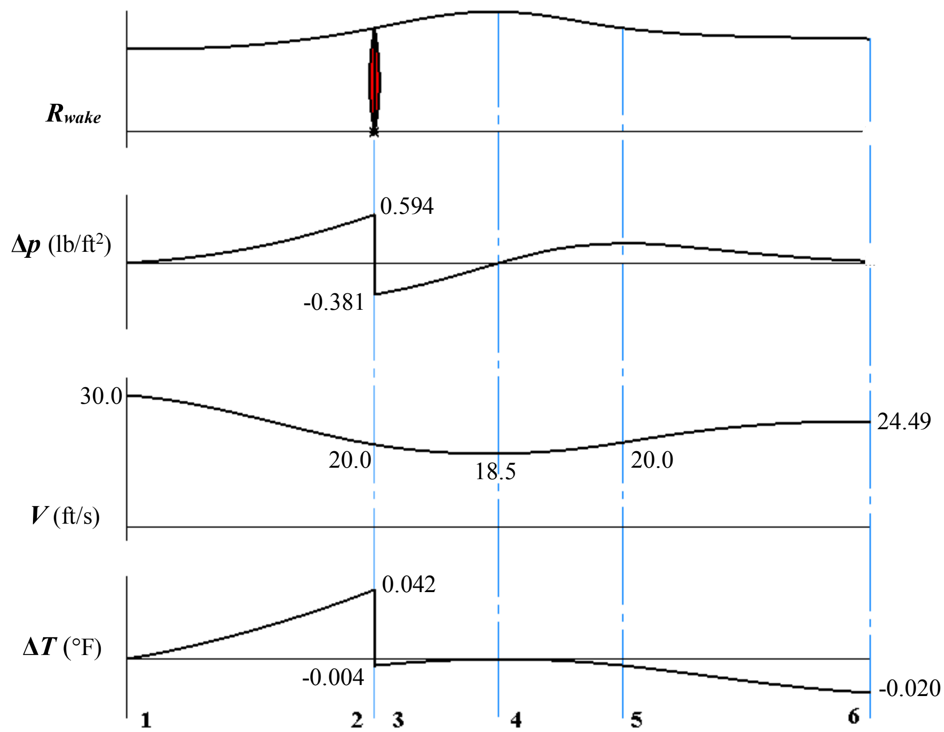

Figure 12 Station Parameter Graphs

It should be noted that in this case a free spinning turbine is raising the air temperature by 0.025°F while the same turbine with a C_P = 0.40 is lowering the temperature by 0.020°F. Implied by this is that turbines operating inefficiently are potentially raising atmospheric temperatures while a properly designed ultra-efficient wind turbine could lower atmospheric temperatures aiding in our fight with global warming.

D. Accelerated Flow Concepts and New Configurations

Accelerated flow concepts can take on different forms of shrouded turbines from diffusers to collectors and concentrators. These may also be referred to as constrained flow versus open rotor or free flow systems. The main idea in the all above is to increase mass flow through the turbine. Most are accepted in theory; few if any have proven to perform superior in practice and none to my knowledge have been commercially successful. For more in depth coverage of this subject matter see Ref. (11) and (12).

The diffuser theories usually use conventional actuator disc concepts with assumed increases in mass flow to evaluate their effectiveness, see Ref. (11). These conventional theories claim that the relative increase in power coefficient will be proportional to the increase in mass flow ratio. The increase in mass flow is often tied to an assumed favorable pressure differential at the aft end of the diffuser or simple concentration from a larger diameter collector. Although these current systems will definitely increase the power coefficient beyond the same open rotor, it is debatable whether or not the *Cp* is any better when the diameter of the diffuser is accounted for or whether the increase in power is more than an equivalent increase in swept area would provide.

In my opinion, one of many reasons why wind turbine diffuser designs don't measure up to expectations is that most make no effort to account for the airflow separation and associated intersection drag that occur naturally between the diffuser ring structure and the blade tips. Intersection drag is a common problem dealt with in aircraft design, but more difficult to deal with in the case of the wind turbine diffuser compounded by the inherent airflow separation and associated adverse pressure gradient which occur at what is the most critical portion of the flow field in the outer radii.

In addition one needs to look at the drag effect of the concentrator and the lift generated by the diffuser. The effect of the diffuser on the flow surrounding the wind turbine it is to both deflect air away from the turbine as well as to slow down the airflow in front of the turbine in reaction to its own lift generation. This becomes another component of the normal force in $F_n V$ causing the change in momentum which increases the internal energy and decreases the kinetic energy, defeating its own purpose. These overall airflow effects alone are surely enough to negate any benefit from the diffusers increase in relative mass flow versus a similar increase of diameter in swept area. Because of this effect on the overall flow field wind tunnel tests of a diffusers cannot accurately simulate the true flow field and resulting effect on mass flow through a turbine unless the scale is very small or tunnel extremely large to eliminate influence from the tunnel walls,

That brings us to the third problem with most diffuser, collector and concentrator designs, they attempt to collect air from outer annular slipstream elements and redirect the airflow inward to be reacted on by smaller turbine radii. As is obvious from Eq. (7) the turbine will be more effective if we move the air flow outward to be reacted on at the largest radius possible.

Most evaluations of concentrator systems ignore the shift in the energy equation from the enthalpy term to the kinetic energy term when airflow is accelerated. When we accelerate the airflow the available kinetic energy increases with the square of the velocity. The power available in the kinetic energy flowing therefore increases once again due to drop in temperature and pressure which must occur. So in effect the available kinetic energy per unit mass reacting with the turbine is greater. Furthermore all the aerodynamic forces reacting with the turbine blades are functions of the dynamic pressure which again is proportional to the square of the velocity. Therefore no simple generalizations can be made for the effect of constrained flow without looking at the correct energy equation.

So notice that I chose to debunk diffuser and concentrator designs but not accelerated flow concepts in general. Accelerating the airflow at the outer radii of the turbine blades while increasing the rotational parameters without decreasing mass flow are the keys to gaining more performance from the next generation of wind turbine. So how do we do this? It takes the combination of three simple devices to make successful use of the accelerated flow concepts. These are two large inner diameter spinner type fairings located fore and aft of the turbine disc and an outer flow control ring. The purpose of the large area forward spinner is to reduce the flow area and therefore accelerate the flow outward to the more effective radii. It was shown the energy extracted is proportional to the square of the radius of the annular elements from which it is extracted and that if the flow is accelerated to these outer radii by a reduction in area then the forces transferring energy to the turbine blades increase with square of the velocity. An added benefit of the accelerated flow is that for the same rotational velocity the flow angles improve to create more torque and less normal force. The purpose of the flow control ring is to prevent radial flow along the turbine blades, eliminate tip losses, accelerate airflow into the turbine wake and deter wake expansion. Unlike a diffuser the flow control ring is made up of an airfoil with its lower high pressure surface to the center of the turbine. In this manner it acts as an airfoil end plate with positive pressure acting to stabilize the flow through the turbine while eliminating losses due to wing tip vortices. At the same time the forced acceleration of the flow downwind of the outer ring has the opposite effect of a diffuser ring or winglet. The flow control ring effect is to accelerate surrounding air into the turbine wake drafting more mass flow through the turbine and not away from the turbine. The aft spinner design assists in further encouraging the wake to contract without a loss in sectional area, while increasing rotational velocity smoothly without separation. This configuration minus tower is depicted in the following Fig. 13.

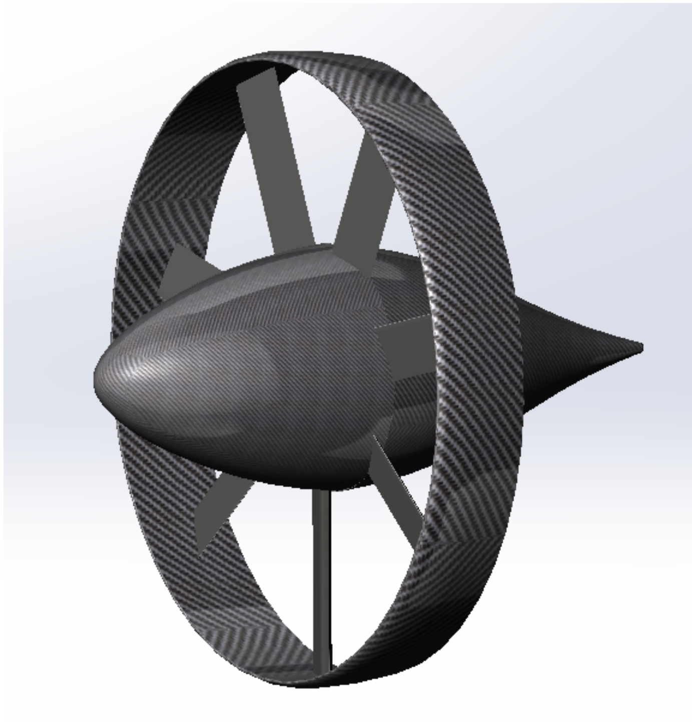

Figure 13 Proposed Thermodynamic Wind Turbine Configuration

Obvious in the above figure is also the return to a multiplicity of rotor blades. Now that the erred Betz limitations have been removed from the equations more power can be extracted cost effectively with additional blades. In addition the new configuration actually will use much shorter smaller blades and therefore less structural material. In general most of the parts of the new design can be manufactured as light weight composite shells without massive spar sections. Blades may even possibly be built lighter and more economical by returning to aluminum skinned sections.

The algorithms show you can't just add more blades to a wind turbine and expect more power when installed on the same generator. Any improvement in wind turbine configuration can only be successful when the blade element parameters are designed in accordance with a corrected design algorithm and the turbine is properly matched to the rotational speed and torque of the electrical generator. Experimentation does demonstrate that turbines will generate higher efficiencies with more blades and higher solidity at lower wind velocities, see any of the research done by Kenneth Visser and others[13]. It requires a generator capable of a higher torque value to absorb the power of the additional blades at the higher freestream velocities. At the same time it takes a delicate balance of pitch angle and blade planform to prevent high axial induction factors.

V. M-BET Algorithms

I have developed two versions of M-BET, one for free flow and one for accelerated flow. The free flow version returns results very close to conventional algorithms for most designs, but as solidity with number of blades increases and rotational velocity decreases M-BET can return results that well exceed the performance of current designs. The program for accelerated flow stands alone in demonstrating the full potential of the next generation of wind turbine. For that reason I will focus this discussion on the M-BET algorithm for accelerated constrained flow.

As with other blade element methods, M-BET discretizes the flow field into annular elements of width *dr*. The MBET design theory assumes that enough blades are present and rotating at a sufficient velocity that the normal force generated by the turbine blades is distributed evenly throughout the turbine disc area as a pressure distribution. One of the major differences between M-BET and conventional BET is that although the blade element forces are calculated independently MBET does not accept that the annular elements are independent of one another. The blade forces must be calculated independently because of the extreme change in relative flow angle from the inner to outer radii. The laminar wake momentum equation was shown to determine the wake profile, but the solution assumed a single force at the center of the overall control volume not annular elements. These annular elements cannot influence themselves upstream independently but interact to influence the overall control volume. It is the sum of the annular element forces that determine the thrust coefficient, inflow and outflow velocity ratios. If the program returns results for annular thrust coefficients $C_{Ti} = F_{ni}/(qA_i)$ that differ extremely, then this would imply possible radial flow from one element to another and a questionable return for the overall results. Blade pitch or chord should be altered until C_{Ti} values for annular elements show reasonably close returns. Alternately velocity and mass flow values would have to be recalculated for each element. At this point in the development of program, radial flow would over complicate the solutions, and I strive for uniform pressure distribution across the turbine disc.

M-BET success is based on applying the principles that I have previously discussed and are implied by Eq. (7), the fundamental $C_{Pi} = 2a_i\lambda_r\lambda_s$. Notably the fact that the energy extracted will increase with the square of the radius of the annular element from which the flow passes through. In addition a_i must be maximized. The solution to a_i was shown in the laminar wake momentum Eq. (42) to be a function of $C_T = f(F_n)$ and therefore we want to minimize any normal force created by the spinner or airfoil shaped ring we create around the turbine. As well it was proposed that if we reduced the area with a large spinner directing flow to the outer radii and constraining it then the kinetic energy available to react with the turbine blades will increase with the square of the velocity.

In order to calculate this new velocity through the turbine, M-BET requires a forward control volume plane at the leading edge of the inlet fairing with area equal to the total turbine area A_2. The aft plane of this control volume is the swept area of the rotor or the overall area minus the spinner area. We will define this area as $A_{2.5}$, conforming to the other defined station positions. The accelerated velocity interacting with the blades thorough this region will therefore be defined as $V_{2.5}$. See Fig. 14 below.

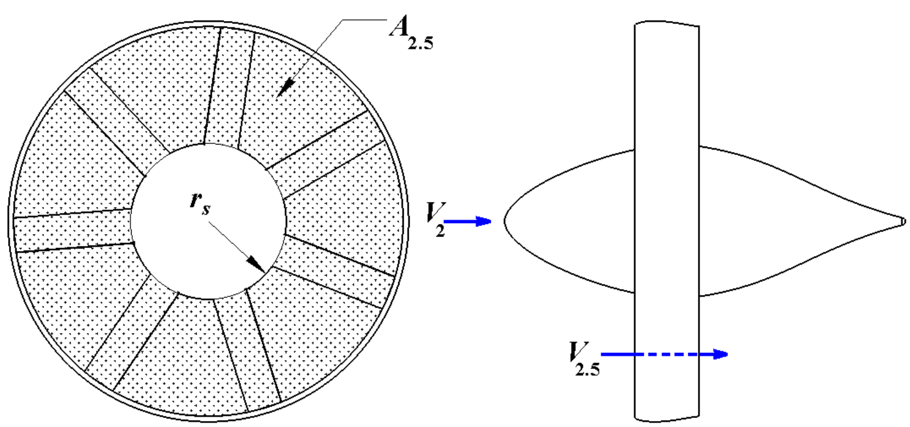

Figure 14 M-BET Turbine Airflow

We define r_s as the radius of the spinner. From conservation of mass flow, $V_2A_2 = V_{2.5}A_{2.5}$. Therefore

$$V_{2.5} = \frac{V_2 A_2}{A_{2.5}} = \frac{V_2 \pi R^2}{\pi\left(R^2 - r_s^2\right)} = \frac{V_2 R^2}{\left(R^2 - r_s^2\right)} \tag{78}$$

or in terms of a_i and the ratio of spinner to turbine radii

$$V_{2.5} = \frac{a_i V_\infty}{1 - \left(r_s/R\right)^2} \tag{79}$$

The relative airflow triangle for M-BET considering constrained flow is simpler than free flow and conventional theory. I do not believe it necessary to calculate the downwash angle of flow due lift and rotation in this case. Most wind tunnel data for airfoils is based on the geometric angle of attack for an airfoil section joined to the side walls of the tunnel in similarly constrained flow. I assume in the program, with our flow accelerated into the constrained ring the change in angle of attack due to induced flow downstream of the turbine will have negligible effect. I also assume tip losses to be negligible due to the end plate effect of the ring. For the free flow program one does need to make allowances for both aspect ratio and tip losses. With that said we can estimate the constrained flow angles and velocities with the following figure 15.

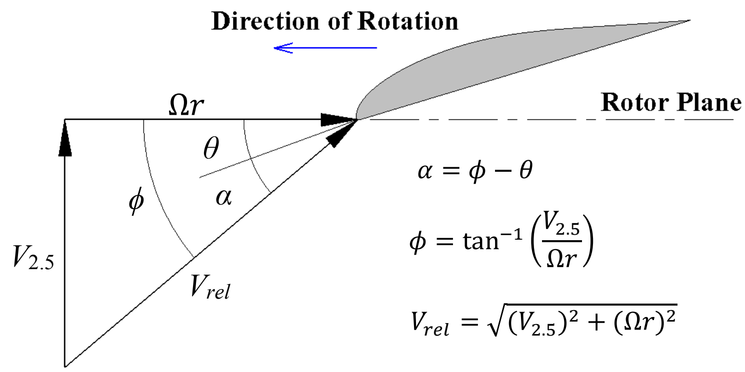

Figure 15 M-BET Airflow Triangle

The M-BET algorithm steps are as follows:

Step1.) Initialize an assumed value for inflow velocity ratio a_i. We derived an optimum from the energy Eq. (56) of $a_{i,optimum} = 0.5 + K/2$. So a good starting point is $a_i = 0.5$ or higher and work the program from there.

Step2.) Calculate $V_{2.5}$ from Eq. (79) for the given choice of spinner radius.

Step 3.) For each annular element of area $A_{2.5}$, layout number of blades, planform chord c, set desired pitch angle θ and Ω. Compute inflow angle ϕ, local angle of attack α and the relative velocity V_{rel} from Fig. 15.

Step 4.) Determine local c_l and c_d from known airfoil data or airfoil design software.

Step 5.) Calculate the normal and tangential forces on each blade element using standard methods of blade element theory such as presented by Hansen.[2]

Step 6.) Sum all the elemental normal forces, F_n = thrust and calculate $C_T = F_n/(qA_2)$. From this calculate the corrected value for a_i from Eq. (43), $a_i = (1 - 0.5C_T)^{2/3}$. Iterate the solution process until the value of a_i is in reasonable agreement.

Step 7.) Calculate $C_{Ti} = F_{ni}/(qA_i)$ for each annular element and confirm the values are reasonably close. If not adjust chord or pitch angle to maintain reasonably uniform pressure differential. Repeat step 6 and 7 until a_i is in reasonable agreement and C_{Ti} represents reasonably uniform flow.

Step 8.) Compute loads on all blades from final elemental normal and tangential forces. Calculate power, thrust, and blade loading again using standard methods.

VI. Conclusion

This paper has laid out a fundamentally revised theory of wind turbine design and analysis using accepted thermodynamic principles in addition to conventional blade element aerodynamic analysis. With respect to conventional theory, previously mistaken limitations have been disproven and removed. The Laminar Wake Momentum Equation is derived with the fundamental new result that $a_i = b_i^2 = (1-0.5C_T)^{2/3}$. The conventional energy equation for a wind turbine is corrected to include thermodynamic terms as it should. A mathematical model for this new thermodynamic wind turbine is presented and the method of extracting thermal energy from the wind turbine is explained. A cost effective ultra-efficient multi-blade turbine which is under development is shown. The new concept uses a large area spinner or fairing to accelerate airflow to the outer radii of the turbine blades. Blade tip losses are eliminated and flow constrained with the use of an outer flow control ring which also is used to accelerate flow in the near wake. An aft spinner encourages smooth wake rotation without separation while the combination discourages wake expansion. These concepts are currently undergoing dynamometer testing for theory validation at Mansberger Aircraft Inc. We hope these new turbines will not only be capable of producing power at efficiencies previously thought impossible but will have the added benefit of reducing atmospheric temperatures and aiding in our ability to deal with global warming.

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About the Author:

Larry Mansberger holds a Bachelor of Science degree in Aerospace Engineering from the University of Texas with post graduate studies in engineering mechanics of composite materials and fluid dynamics. He has additional formal training in advanced composite design, fabrication and repair. He also holds FAA Certified Airframe and Powerplant Mechanic Licenses with FAA Inspector Authorizations. He has been flying since 1976 and is a current licensed private pilot with single engine land, seaplane and glider ratings. His 40 plus years of experience includes successfully designing, building and flying his own original design, all composite, multi engine, canard aircraft. Mansberger Aircraft has been in the business of providing specialty composite airframe services since 1985. This paper and other current endeavors focus on applying Mansberger Aircraft's unique skill set to wind energy research and product development. For more information visit www.MansbergerAircraft.com *or for correspondence with respect to this paper contact the author at* wind.turbine.mai@gmail.com.